<Chapter 2>

Basic Theory of the Lotka-Volterra Competition Model

In the last chapter, we focused on the logistic model, that is based on how species of organisms naturally multiply in environments, but it is important to consider that there is never only one species of organism in a given environment. There are other species of organism that eat the same food, and this affects each organism through competition. But what about in business? Unless your company is the only one in the industry, there are competitors whose presence can affect your growth. When there are several players playing the same game, there will always be a winner and a loser. In the business world, wins and losses come in the form of market share, differences in profitability and withdrawals from the market.

Even in biology, the effect that two species of organism have on each other is called *"competition"*. There is a numerical model that represents this model very well - the simplest of which is called the "*Lotka-Volterra Competition Model"*. The model is shown below in terms of ***x*** and ***y***.

$$\left.\begin{aligned} dx/dt &= r_1x(1-(x+by)/K_1) \\ dy/dt &= r_2y(1-(ax+y)/K_2) \end{aligned}\right\} \quad (2.1)$$

There are two equations to see the competition between ***x*** and ***y***. The content of the equation is quite similar to the logistic equation *(1.3)*, but there is a difference in the numerator in ***x/K***. ***a*** is the degree of effect that ***x*** has on ***y***'s multiplication, and ***b*** is the degree of effect that ***y*** has on ***x***'s multiplication. If both ***a*** and ***b*** are positive values, then the numerator will become bigger than the denominator ***K*** where there was only one species. As the numerator increases, the value of $(1-(x+by)/K_1)$ and $(1-(ax+y)/K_2)$ will decrease, as will the rate of change of multiplication, *dx/dt* and *dy/dt*. In other words, the growth of ***x*** and ***y*** are limited by the presence of competitors. If *a=0* and *b=0*, it will revert back to the original formula for one species *(1.3)*.

Different to when there was only one species, there are now six parameters, r_1, r_2, ***a, b,*** K_1, ***and*** K_2. The rate of change will depend on how these numbers are put together, so it can be predicted that there will be many patterns of growth of the whole values ***x*** and ***y***. We were able to analytically solve the logistic equation *(1.3)* into a *"number of individuals = x"*, *"time = t"* function like *(1.4)*, but it is unfortunately not possible to do this with *(2.1)*, as there is no analytical solution. However, our goal is to see how the values of ***x*** and ***y*** increase (or decrease) over time. We can still do a numerical calculation, so let's set a coefficient and try it out.

Let's set arbitrary values for the coefficients *(Table 3)*. We can set a value for the initial value by randomly combining the seven values *(Table 4).* Next, plot the change in ***x*** and ***y*** values over time by gradually increasing the value of the time *(⊿t)*. In curve ①, ***x*** has an initial value *(t=0)* of *10*, and ***y*** has an initial value *(t=0)* of *98*. In this case, it is showing what happens to the combination of ***x*** and ***y*** as the time increases *(t=1,2,3,4…)*. The arrow shows the direction in which it is moving. *Figure*

13-1 shows the results of calculating all the seven combinations created with arbitrary conditions.

The horizontal axis is the number of individuals of ***x***, and the vertical axis is the number of individuals of ***y***. It is interesting that regardless of the combination of initial values from which the line starts, the lines end up in the same area. ***x*** and ***y*** are living in competition over the same food source, but as time passes, eventually the situation settles, with a certain combination of individuals living together. Therefore we can see that from that point onwards, the two species will be able to live amicably together. In biology, when a system enters a certain condition, and it stays in that condition, that condition is known as a *state of equilibrium*.

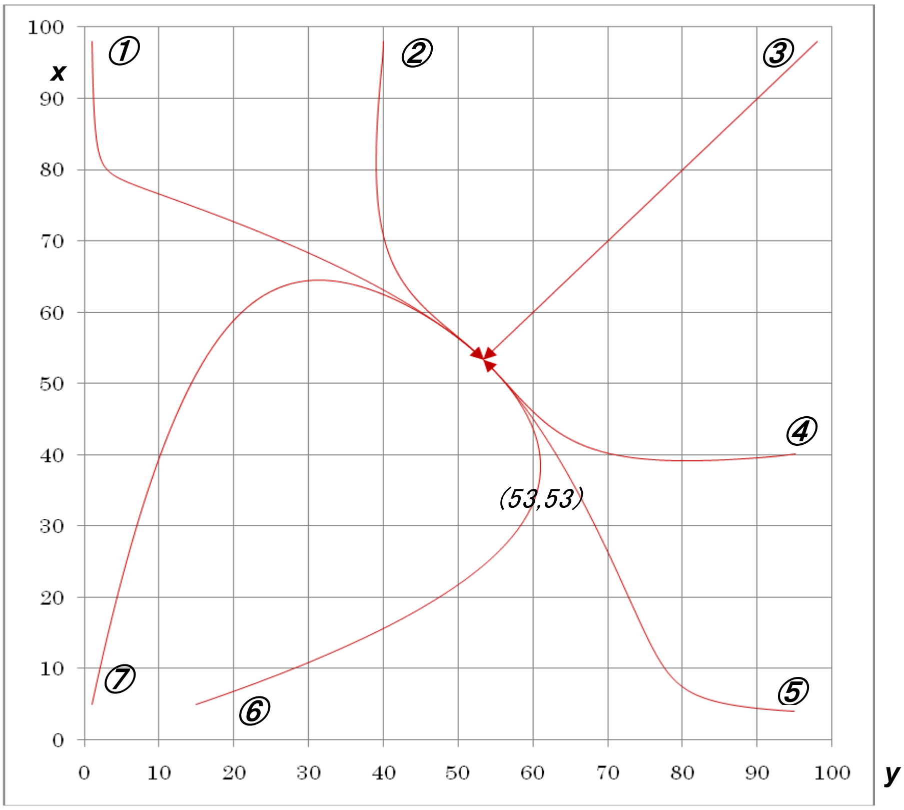
Table 3 - Arbitrary coefficients

x	r_1	*0.5*	K_1	*80*	*a*	*0.5*
y	r_2	*0.5*	K_2	*80*	*b*	*0.5*

Table 4 - Arbitrary initial values

Curve	①	②	③	④	⑤	⑥	⑦
x	*10*	*40*	*98*	*95*	*95*	*15*	*2*
y	*98*	*98*	*98*	*40*	*4*	*5*	*5*

Figure 13-1

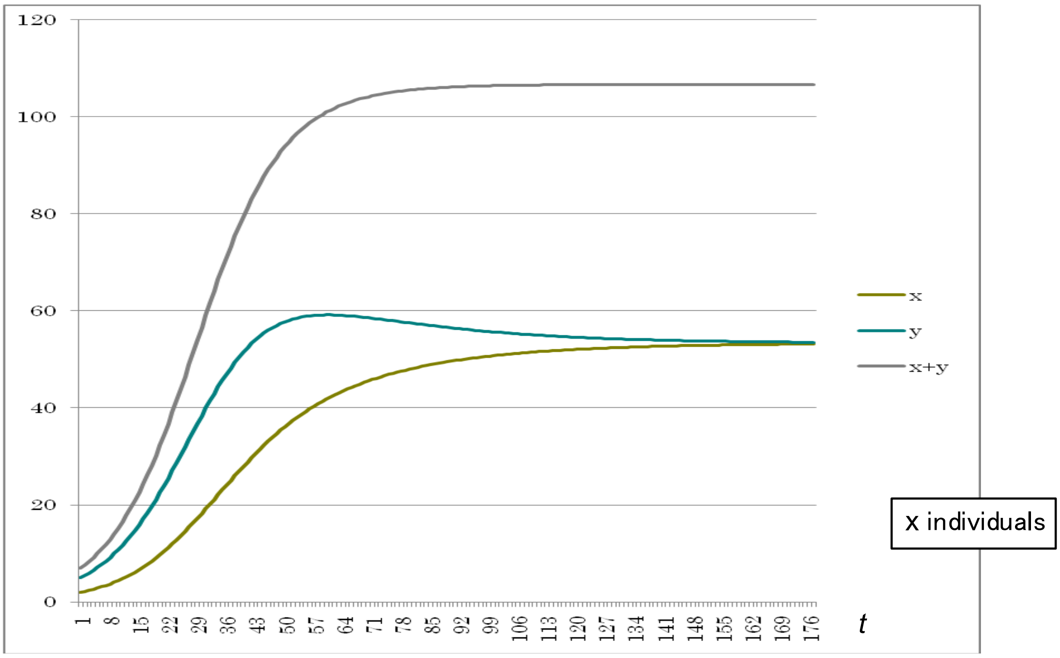


⊿t=0.2

In *Figure 13-1* seven *curves* represent the changes in values of ***x*** and ***y*** over time. *Figure 13-2* below represents the curve ⑦ by the respective curves. There are differences in the value of ***x*** and ***y***, but it is clear that the total value of *x+y* shows logistic growth that is identical to that of a species of organism. Additionally, it is interesting that the carrying capacity ***K*** is *80* for both ***x*** and ***y***. The fact that they are both in competition and limiting each other's growth means that the total carrying capacity for both species *160 (=80+80)* is less than it would be if there was just one species multiplying.

Figure 13-2

< Number of individuals with the initial value ⑦>



Let's try applying this model to the sales of two competing companies, ***x*** and ***y***. ***r*** is each company's growth coefficient, ***K*** is each company's carrying capacity, and the competition coefficients ***a*** and ***b*** represent the effect that each company has on the other's growth.

＜When the growth coefficient is different＞

Let's try changing the growth coefficient ***r***. If we reduce ***y***'s growth coefficient r_2 by *0.3* like shown in *Table 5*, *Figure 13-1* changes shape and becomes like *Figure 13-3.* The lines change slightly, but the point of coexistence doesn't change. The point of coexistence (or equilibrium) is the point where the growth (increase/decrease) stops for both ***x*** and ***y***. In other words, it is the point when the change in growth *(2.1)* becomes zero. The point where *(2.1)* becomes zero is either where the growth coefficients r_1 and r_2 become *zero*, or where $(1-(x+by)/K_1)$ and $(1-(ax+y)/K_2)$ become *zero*. In the last case, it can be written as below.

$$\left.\begin{aligned} 1-(x+by)/K_1 &= 0 \\ 1-(ax+y)/K_2 &= 0 \end{aligned}\right\} \quad (2.2)$$

And rewritten with ***y***, it would be…

$$\left.\begin{aligned} y &= (K_1-x)/b \\ y &= K_2-ax \end{aligned}\right\} \quad (2.3)$$

y is the linear function (straight line) of ***x***. If we transform *(2.3)*, it becomes…

$$\left.\begin{aligned} x &= K_1 - by \\ y &= -ax + K_2 \end{aligned}\right\} \quad (2.4)$$

The two straight lines pass through $x = K_1$ when $y=0$ and $y = K_2$ when $x=0$ (Blue and green lines in *Figure 13-3*)

The coordinates *(2.2)* of these straight lines represent the point when ***x*** and ***y***'s growth stops. The intersection of both lines is the point when they both stop growing - the point of coexistence. The coordinates of the intersection *(x,y)* can be found by solving two simultaneous equations as shown below.

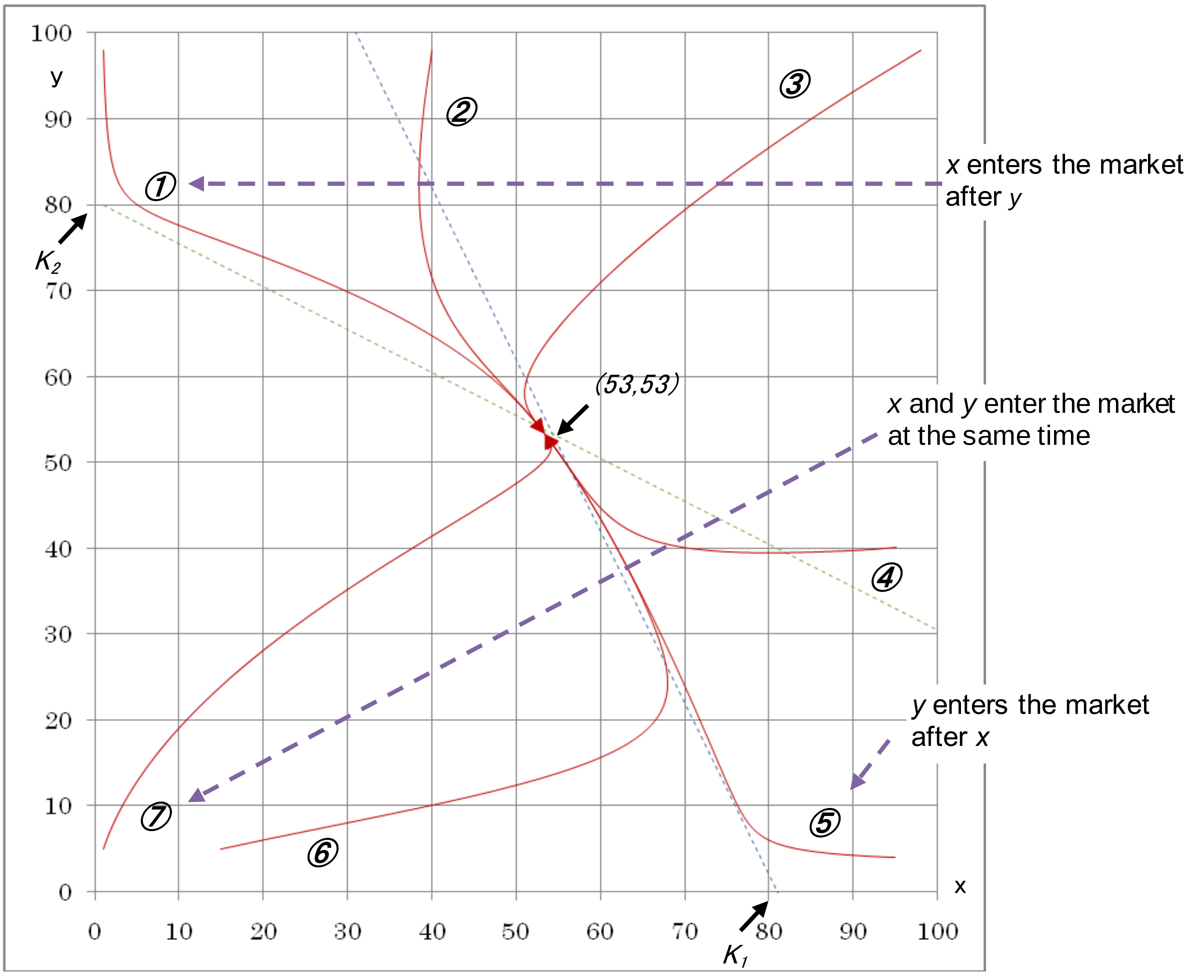
$$\left.\begin{aligned} x &= (K_1-bK_2)/(1-ab) \\ y &= (K_2-aK_1)/(1-ab) \end{aligned}\right\} \quad (2.5)$$

Here, it is important to note that the intersection coordinates do not include the growth coefficient ***r***. In management terms, stable market share is decided only by the carrying capacity (management resources) and the competition coefficient.

Table 5

	Growth Coefficient		*Environmental Capacity*		*Competition Coefficient*	
x	r_1	*0.5*	K_1	*80*	*a*	*0.5*
y	r_2	***0.3***	K_2	*80*	*b*	*0.5*

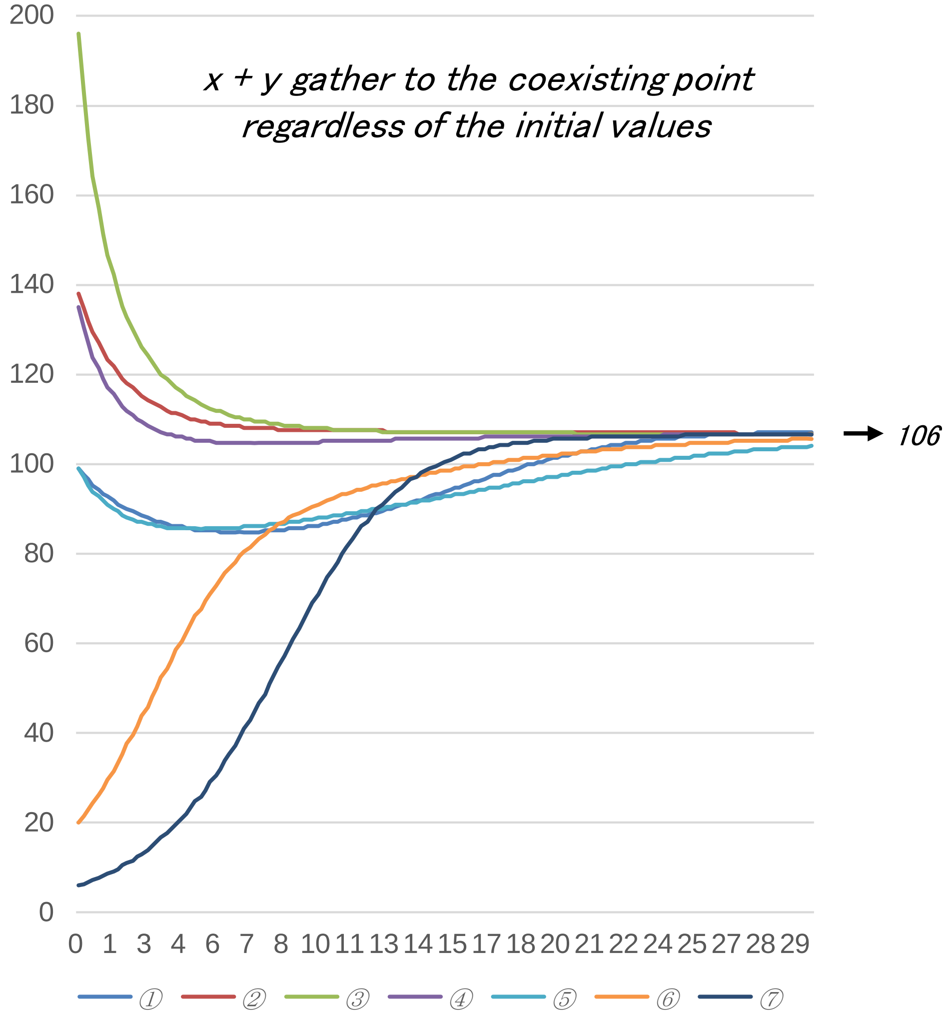
Figure 13-3



r_1=0.5, r_2=0.3 K_1=80, K_2=80 a=0.5, b=0.5

The sales of both companies stop at *53* - lower than the level that they would be expected to reach if they had no competitors - *80*. However, the total market size is *106*, which is more than 30% higher than it would have been with only one company. If one company manages to force the other to leave the market, supply will be limited to the remaining company. Therefore, the market size is bigger when two companies coexist. *Figure 14* represents the total market supply as time progresses, with the total supply of companies ***x*** and ***y*** on the vertical axis. This makes it clear that in all cases, total supply gathers towards the point of coexistence, *106 (=53+53).*

Figure 14



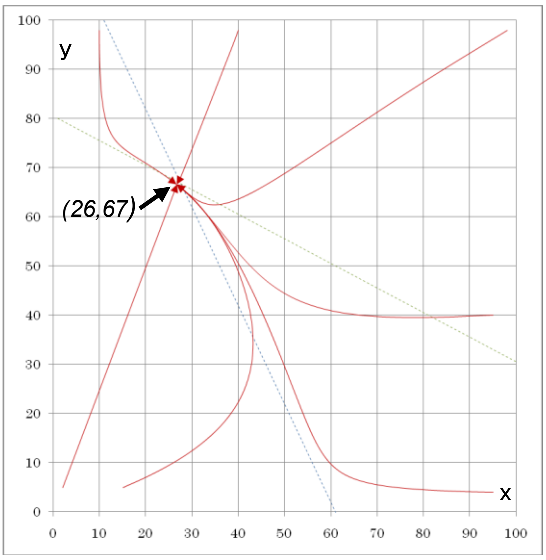
＜When the carrying capacity is different＞

Next, let's make changes to the carrying capacity. If we decrease the carrying capacity of ***x*** by half as shown in *Table 6* below, the results, as shown in *Figure 15-1*, would mean that ***y***, which wins by volume, would be the only company to survive. However, if we decrease the difference in capacity, both companies can coexist. For example, if $K_1=60$ and $K_2=80$, the point of intersection would be *(27,67)* and both companies can coexist, despite ***y*** retaining a higher capacity. (*Figure 15-2*）

Table 6

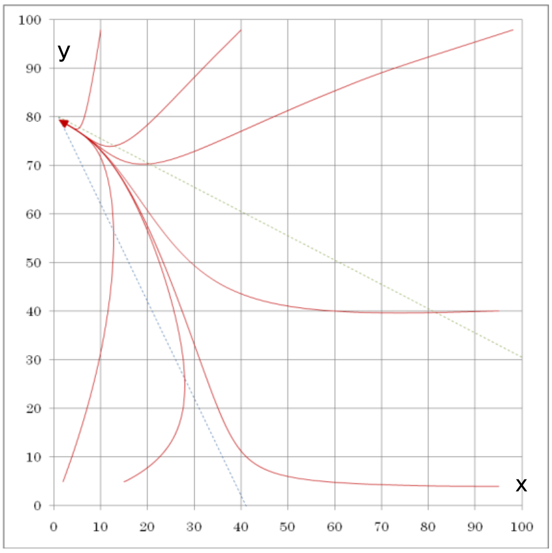
	Growth Coefficient		Environmental Capacity		Competition Coefficient	
x	r_1	0.5	K_1	**40**	a	0.5
y	r_2	0.5	K_2	80	b	0.5

Figure 15-1



$r_1=0.5$, $r_2=0.5$ $K_1=40$, $K_2=80$ $a=0.5$, $b=0.5$

Figure 15-2



$r_1=0.5$, $r_2=0.5$ $K_1=60$, $K_2=80$ $a=0.5$, $b=0.5$

＜When the competition coefficient is different＞

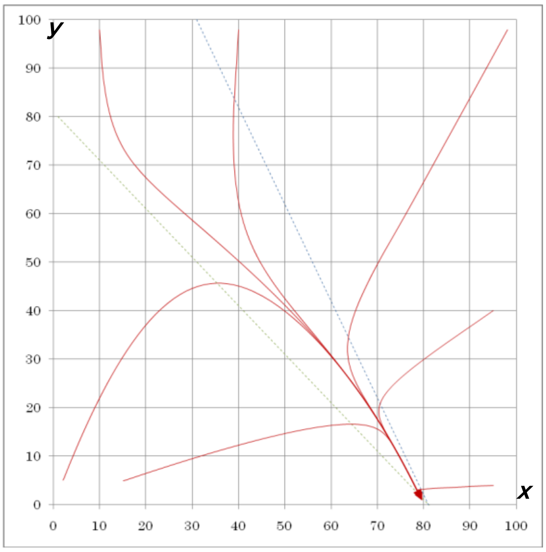
In case of "0<ab < 1"

Next, let's try only changing the competition coefficient of ***x*** from *0.5* to *1.0 (Table 7)*. The results, as shown in *Figure 16-1*, show that ***y*** is selected off due to the increased influence of *x*. In business, this can happen when one business offers much greater value for customers, such as when company ***x*** decreases their pricing and ***y*** fails to do the same, or if ***x***'s products are much higher quality than ***y***'s etc. Here, businesses "selection" doesn't necessarily refer to bankruptcy, but can also refer to when they decide to pull out of the market. There are many cases in which a certain product or service doesn't meet expectations and is withdrawn, but the company continues to operate.

Table 7

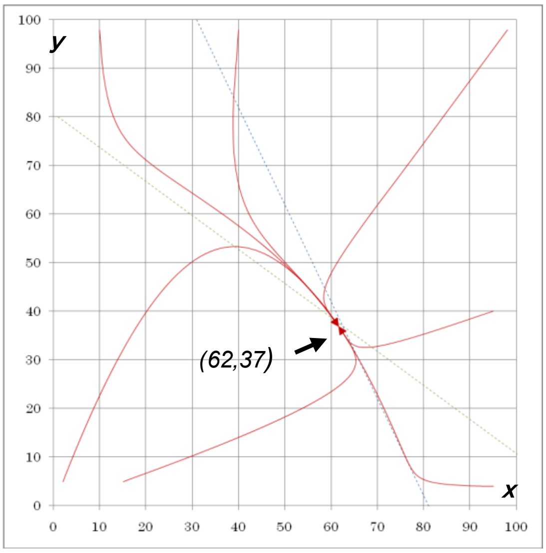
	Growth Coefficient		*Carrying Capacity*		*Competition Coefficient*	
x	r_1	*0.5*	K_1	*80*	*a*	***1.0***
y	r_2	*0.5*	K_2	*80*	*b*	*0.5*
					axb	*0.5*

Figure 16-1



r1=0.5, r2=0.5 K1=80, K2=80, a=1.0, b=0.5

Figure 16-2



r1=0.5, r2=0.5 K1=80, K2=80, a=0.7, b=0.5

So what happens after mild strategy changes that don't have such a significant impact on the company's value in the eyes of the customer? If we reduce the competition coefficient of ***x*** to *0.7*, ***y*** won't be forced to be selected. The point of intersection for this case is *(62,37)*, and both companies can coexist, with ***x***, that has better strategy, with the larger share. *(Figure 16-2)*. This sort of situation could occur, for example, when both companies have similar pricing, but ***x*** has better design or after sales service etc. compared to ***y***.

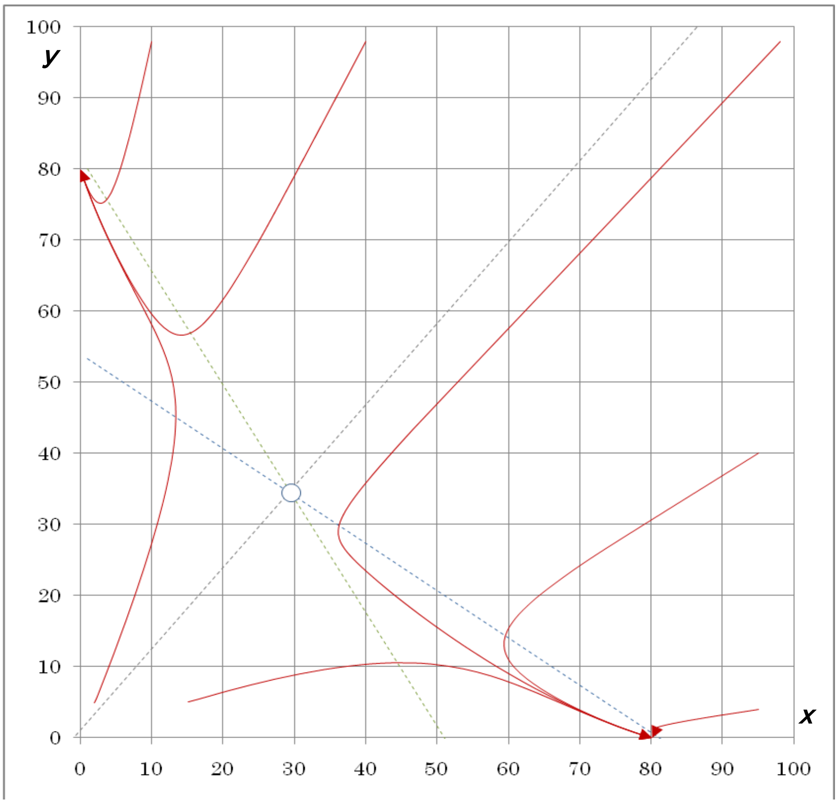
In case of a*b > 1

What happens if both companies show innovative strategies and put up a strong fight? *(Table 8)* Interestingly, in this case the winner is decided by the initial value. In all the other scenarios, the companies eventually reached the same point regardless of the initial value, but when the competition coefficient *a*b > 1,* the fate of the companies depends on the initial value. *(Figure 17)*

Table 8

	Growth Coefficient		*Carrying Capacity*		*Competition Coefficient*	
x	*r1*	*0.5*	*K1*	*80*	*a*	***1.6***
y	*r2*	*0.5*	*K2*	*80*	*b*	***1.5***
					axb	*2.4*

Figure 17



r_1=0.5, r_2=0.5 K_1=80, K_2=80   a=1.6, b=1.5

When both companies start to implement strategies that have large impacts on the other, they both experience great instability. The results of these calculations reflect the instability that companies in this situation experience in real life. With regards to the initial value having a significant effect, it can be seen that companies with smaller sales revenue tend to use more extreme strategies to outdo their competitors, and this can significantly increase the possibility that the company will self-destruct. Most business people will be familiar with these concepts from common sense and experience, but it is interesting that they can also be proven mathematically.

In the next chapter, the model's theory will be applied directly to real life management situations, to allow you to predict the future of your company.

<Chapter 3>

Applying the Model to Management - Predicting Future Competition

As seen in *Chapter 2*, with the *Lotka-Volterra Competition Model*, the competition results are largely affected by the *competition coefficient (a,b)* and the *carrying capacity (K)*. In this chapter, real examples will be used to set the coefficients and predict the future success of a business.

Figure 18 and *Table 9* show the relationship between sales and profits of 9 companies in the intermediate bulk containers industry in Japan, separated into different Strategy Groups with *Michael Porter*'s theory. Companies in this industry can be separated into four groups, *A* to *D*, along the two axis of sales method and product structure. *a,b,c*… represent the suppliers that belong in each group. Most suppliers make outright sales of tank type IBC (*Group A*), and there are few that offer liner (inner bag) type IBCs (*Group C,D*). Companies that offer rentals (*Group D*) have a high entry barrier as it requires both financial and operational strength and competence.

According to Porter's competitive strategy theory, profitability increases across Strategy Groups in the order of A → C → D, with D, which has the highest entry barrier, having the most effective differentiation strategy. Each supplier's profitability shown in *Table 9* is also connected to this theory.

Figure 18

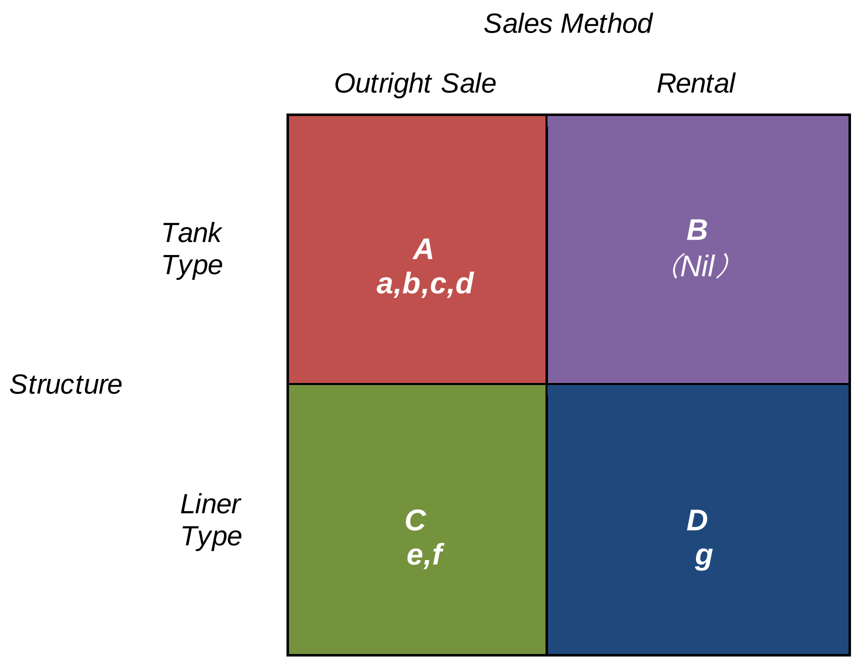


Table 9

＜Sales revenue and Operating profits of IBC suppliers (2001) ＞

Group		Revenue (JPY million)	Market share	EBIT	%
A	a	900	32%	36	4%
	b	900	32%	120	13%
	c	750	26%	120	16%
	d	300	11%	-125	-42%
		2,850	100%	151	5%
B	e	520	81%	120	23%
	f	120	19%	26	22%
		640	100%	146	23%
D	g	670	105%	178	27%

＜Calculating the Competition Coefficient＞

In order to put the competition model to use in real business cases, there is a need to derive the competition coefficient. But is it possible to express the degree of competition numerically? When one species of organism is in competition with another, it is usually over a food source. The more similar the size of the organism, the more intense the fight to survive on one food source becomes. It is possible to make parallels to business. It is easy to imagine that your company could get into intense competition with a competitor over the same customers if you were both selling similar products.

Figure 19 shows the sales of three companies (vertical axis) and the market that they target (horizontal axis) as standard distributions. Here we can see that the intensity of the competition between the companies corresponds to the degree their distributions overlap. The effect that ***y*** has on ***x*** (the competition coefficient) is the percentage **“*Overlap* o_1”** of the “area of the ***x*** distribution”, and the effect that ***x*** has on ***y*** is the percentage **“*Overlap* o_1”** of the “area of the ***y*** distribution”. In the same way, the effect that ***y*** has on ***z*** is the percentage **“*Overlap* o_2”** of the “area of the ***z*** distribution”.

Figure 19

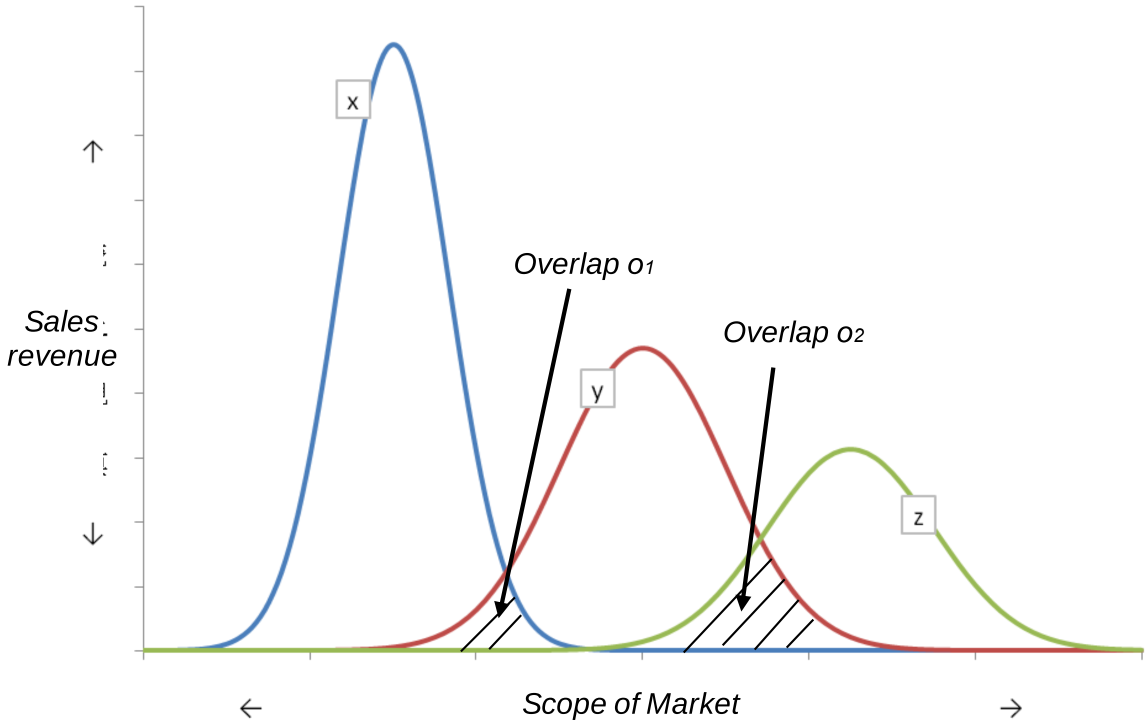


Table 10 shows the sales of each IBCs supplier, separated by the products. *1-6* are chemicals, *7-13* are food products and *14-16* are cosmetics. Each supplier targets different markets based on the unique characteristics of their containers and services, but there is a lot of overlap.

Table 10

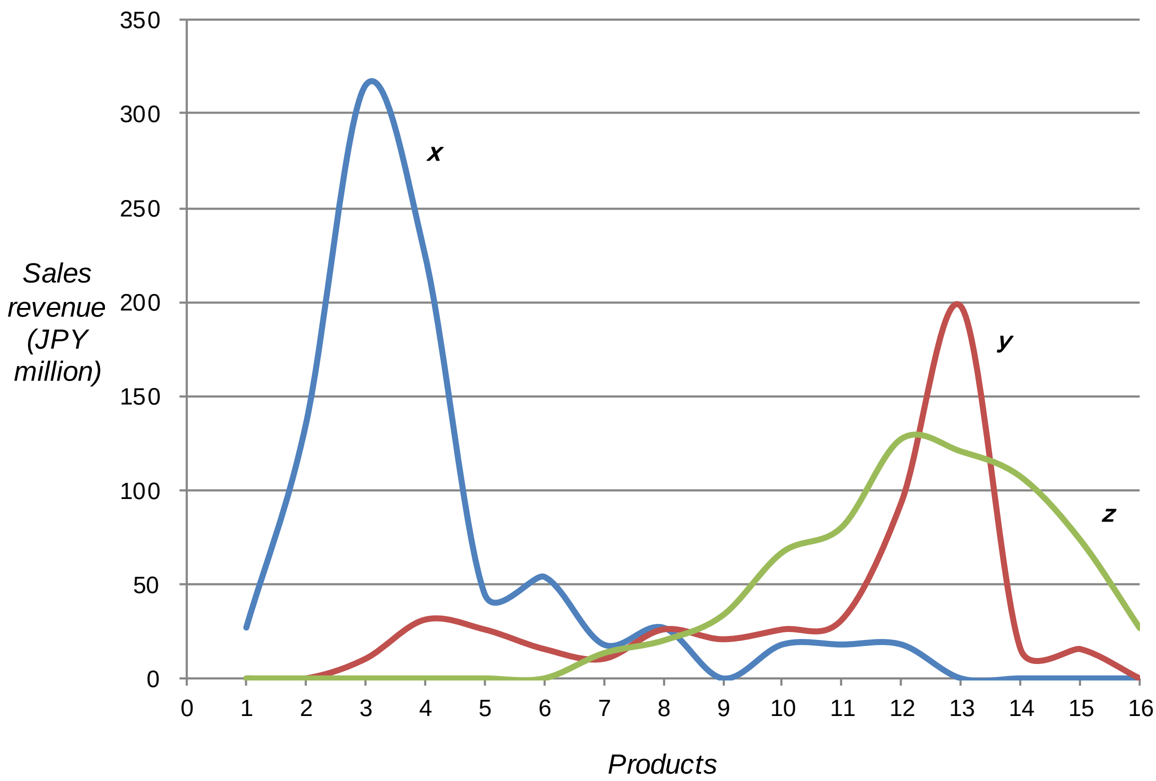
Million yen

			Supplier			Supplier		
Industry		*Product*	*x*	*y*	*z*	*x*	*y*	*z*
Chemicals	*1*	*Ink*	*3%*	*0%*	*0%*	*27*	*0*	*0*
	2	*Paint*	*15%*	*0%*	*0%*	*135*	*0*	*0*
	3	*Glue*	*35%*	*2%*	*0%*	*315*	*10*	*0*
	4	*Emulsion*	*25%*	*6%*	*0%*	*225*	*31*	*0*
	5	*Lubricant*	*5%*	*5%*	*0%*	*45*	*26*	*0*
	6	*Other*	*6%*	*3%*	*0%*	*54*	*16*	*0*
Food	*7*	*Other*	*2%*	*2%*	*2%*	*18*	*10*	*13*
	8	*Sauce*	*3%*	*5%*	*3%*	*27*	*26*	*20*
	9	*Raw Egg*	*0%*	*4%*	*5%*	*0*	*21*	*34*

	10	*Sauce*	*2%*	*5%*	*10%*	*18*	*26*	*67*
	11	*Pulp*	*2%*	*6%*	*12%*	*18*	*31*	*80*
	12	*Edible Oil*	*2%*	*18%*	*19%*	*18*	*94*	*127*
	13	*Dairy*	*0%*	*38%*	*18%*	*0*	*198*	*121*
Cosmetics	*14*	*Haircare*	*0%*	*3%*	*16%*	*0*	*16*	*107*
	15	*Skincare*	*0%*	*3%*	*11%*	*0*	*16*	*74*
	16	*Others*	*0%*	*0%*	*4%*	*0*	*0*	*27*
			100%	*100%*	*100%*	*900*	*520*	*670*

Figure 20

Representation of the overlap of each supplier: compatible of Table 10



If we consider the overlapping areas to be the effect that each supplier has on others, the competition coefficients would be as below. (*Table 11)*.

Table 11

	Million yen	
Overlap between x and y	*174*	··①
Overlap between z and x	*88*	··②
Overlap between z and y	*354*	··③
Area of x	*900*	··④
Area of y	*520*	··⑤
Area of z	*670*	··⑥

Impact of one supplier on another	*Competition Coefficient*	
y→x	*0.19 =α*	··①/④
z→x	*0.10 =β*	··②/④
x→y	*0.33 =ε*	··①/⑤
z→y	*0.68 =γ*	··③/⑤
x→z	*0.13 =λ*	··②/⑥
y→z	*0.53 =μ*	··③/⑥

The growth coefficient ***r*** can be calculated based on the number of years since entering the market as *0.2* for ***x***, *0.4* for ***y*** and *0.5* for ***z***. Refer to the <Growth Coefficient ***r***> in the first chapter.

The carrying capacity ***K*** should be replaced by the management vision. This is because despite the size of the potential market, companies do not grow larger than the manager's vision. Despite ***x*** being the industry leader, management was only focused on the domestic market, resulting in K_2 manager. ***y*** and ***z*** had expanding vision to overseas markets, resulting in K_3. Managers. Refer to <*Environmental Capacity K*> in *Chapter 1.*

Table 12

Supplier	*Growth Coefficient*		*Management Vision*		*Initial Value (million yen)*
x	r_1	*0.2*	K_1	*1,000*	*900*
y	r_2	*0.4*	K_2	*1,500*	*520*
z	r_3	*0.5*	K_3	*1,500*	*670*

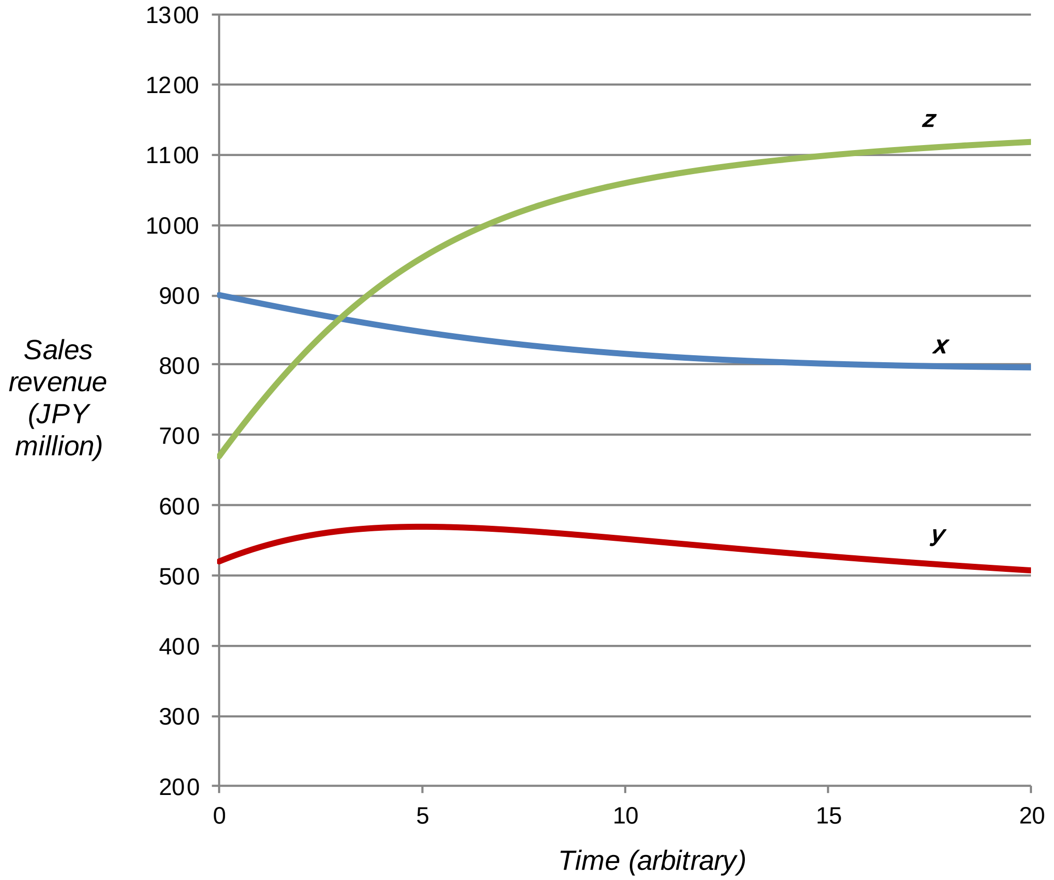
＜*Predicting the future of the three companies*＞

We now have all the necessary parameters to do the equations. The rate of change of growth of the three companies can be calculated using the theory from *Chapter 2*. Below shows the information for each company substituted into the differential equation *(3.1)*.

$$\left.\begin{aligned} dx/dt &= r_1 x\,(1-(x+\alpha y+\beta z)/K_1) \\ dy/dt &= r_2 y\,(1-(\varepsilon x+y+\gamma z)/K_2) \\ dz/dt &= r_3 z\,(1-(\lambda x+\mu y+z)/K_3) \end{aligned}\right\} \qquad (3.1)$$

Figure 22 shows the calculation results of substituting all the parameters into *(3.1)*. It can be seen from the results that the sales of supplier ***z*** are expected to increase, while ***x*** gradually degreases the sales revenue, and ***y***'s increase turns to decrease after some time as a result of competition with ***x*** and ***z***.

Figure 22



The reason that ***z*** continues to grow while ***y*** gradually decreases, despite ***K*** being the same (*Figure 12*), can be attributed to the difference in competition coefficient. The effect that ***z*** has on ***y*** is *0.68*, and the effect that ***y*** has on ***z*** is *0.53* - this is not a big difference. The difference appears when we consider the effect of ***x***. The effect of ***x*** on ***y*** is *0.33*, which is around three times the effect that ***x*** has on ***z***, which is *0.13*. As of 2011, 10 years after the original values were taken, surveys have found the actual performance to be almost identical to these calculations.

To end this chapter, let's sum up the meaning and uses of the model.

<u>Allows you to mathematically represent differentiation</u>

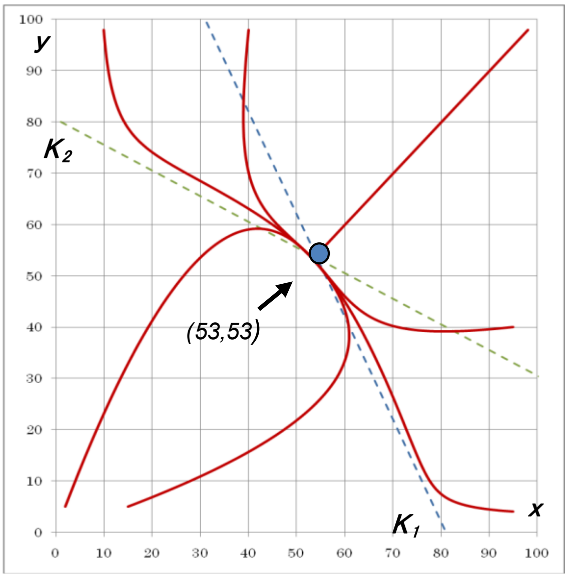
The competition model (*Formula 2.1*) shows how the competition coefficient has a significant effect on the growth of each company. Generally speaking, if your company affects another, you will also be affected by them. Competition with competitors is represented by overlaps in target customers, which is represented as the competition coefficient. It is possible to accurately set the competition coefficient by searching for information about your competitors. Mathematically representing your differentiation allows you to verify your company's strategy from a detached perspective.

<u>Allows you not to underestimate the competitions</u>

At this point, it is important to note how this competition model offers an important warning to management that is focused on maintaining their business's current position. As shown in this basic theory of *Chapter 2*, when two companies with a competition coefficient of *0.5* compete in a market with an environmental capacity of *80*, their point of coexistence is (53,53). (*Figure 23-1)*. If we think of the carrying capacity as sales volume, we can see that company ***x*** has the potential to reach a sales level of *80*, but can only reach *53* due to the competition of ***y***. In this case, if the management of company ***x*** mistakenly decide to focus on simply maintaining sales at the level of ***53***, the company's carrying capacity instantly drops from *80* to *53*. In this case, the company will end up having to compete with a carrying capacity of *53* against company ***y***, whose capacity remains at *80*. So what happens in this case? The point of coexistence changes significantly from *(53,53)* to *(17,71)*. (*Figure 23-2*) It is evident that from then onwards, ***x*** begins to decline and eventually disappears. (*Figure 23-3*).

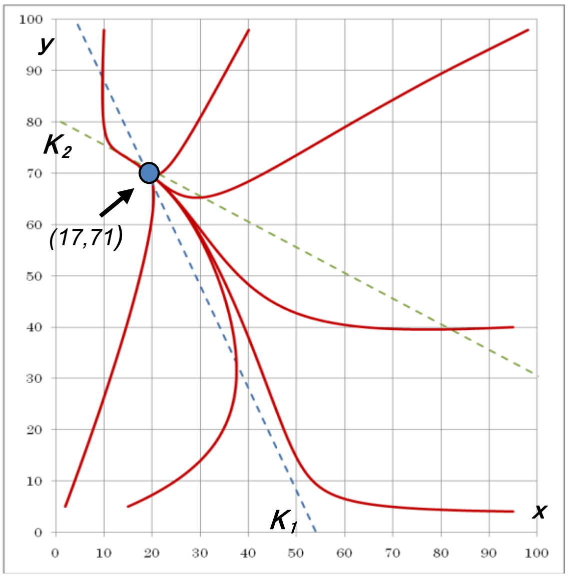
In order to maintain a sales volume of *53*, it is necessary to aim for sales of *80*, not *53*. The model makes it clear that misunderstanding this concept can be fatal. This also needs to be taken into account when creating budgets. This reflects how dangerous it is to have business energy decrease without realizing it until it is too late.

Figure 23-1



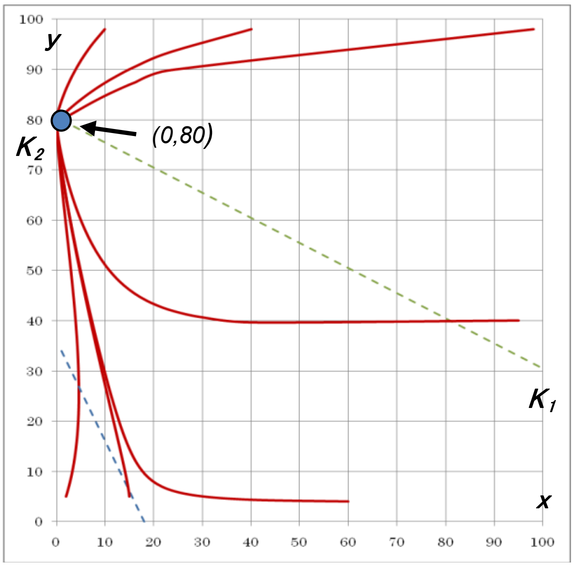
Competition under (80,80) results in (53,53) (r_1,r_2=0.5, a,b=0.5)

Figure 23-2



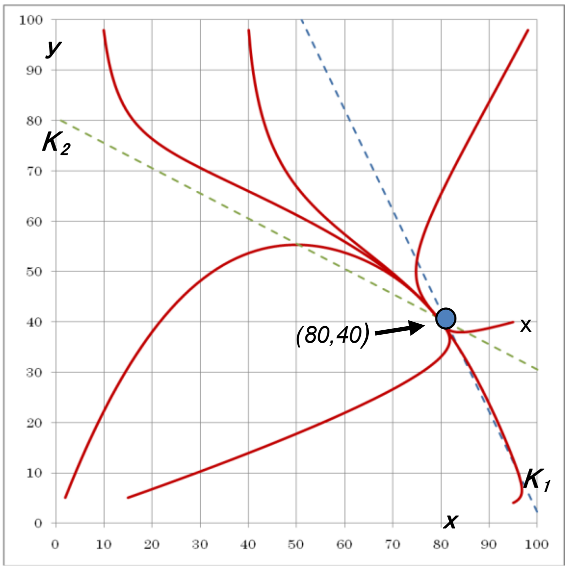
Competition under (53,80) results in (17,71) (r_1,r_2=0.5, a,b=0.5)

Figure 23-3



Competition under (17,71) results with x losing out to y (0,80)

Figure 23-4



To aim for (80,40), for example, x's K_1 must be 100. (r_1,r_2=0.5, a,b=0.5)

Next, let's see what carrying capacity of ***x*** (*K_1*) is required to change the point of coexistence from *(53,53)* to *(80,40)* with ***x*** in the lead. If we gradually increase the ***K_1*** (move the blue dotted line to the right in parallel) and search for the appropriate coexistence coordinates, we can find that the necessary carrying capacity of ***x*** is *100*. In other words, the business must devote all of its energy (management focus) to the goal of *100*, which is *25%* higher than *80*. (*Figure 23-4*)

As this example illustrates, the model is a good way to mathematically and visually represent your business's situation, and it makes it clear where management should be visionary in the future to achieve desired results.

<u>Allows you to predict the future - which gives you the opportunity to change it</u>

What should management do when they predict their company's future and become aware of their relationships with their competitors? Should they just accept the results of the calculation as what their business will be like in the future? No, the results should prompt them to think what they can do for a better future. In order to change the results of the calculation, the conditions and premises on which the calculation stands must be changed. But what should be changed, and how will the results be affected? This mathematical model allows easy simulation, and it is essential that management experience the process of using the model themselves.

<u>Allows you to understand what needs to be done to change the future</u>

In order to accurately predict the future of your company based on its competition with other companies, it is necessary to gain accurate information (on management vision, competitive strategy, etc.) of both your own company, and its competitors. In other words, it allows you to reconsider your company's position in the market and research your competitors. Sun Tzu says in his "The Art of War" - "If you know the enemy and know yourself, you need not fear the result of a hundred battles."

The end of Chapter 3